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Dimensionless Coupling of Superstrings to Supersymmetric Gauge Theories and Scale Invariant Superstring Actions

M. Awada*and F. Mansouri†

Physics Department, University of Cincinnati, Cincinnati, OH 45221

Abstract

We construct new superstring actions which are distinguished from the standard superstrings by being space-time scale invariant. Like standard superstrings, they are also reparametrization invariant, space-time supersymmetric, and invariant under local scale transformations of the world sheet. We discuss two possible scenarios in which these actions could play a significant role, in particular one which involves their coupling to supersymmetric gauge theories.

*e-mail address: moustafa@physunc.phy.uc.edu

†e-mail address: mansouri@uc.edu

It is well known that string models were the outgrowth of the attempts to formulate a theory of strong interactions. Early work on these models made it clear that standard string theories are not well suited for this purpose. In particular, the string based scattering amplitudes at high energy and fixed scattering angle fall off exponentially with energy, whereas experiments in the deep inelastic region indicate that they should be power behaved. This was, in fact, one of the primary reasons for reinterpreting strings as theories incorporating all interactions in the hope that the connection to QCD would emerge with further study.

The power law behaviour of the scattering amplitudes suggests that a string theory consistent with QCD at short distances must have a phase which is characterized by long range order i.e., by absence of a scale. To this end, Polyakov [1] considered modifying the Nambu action by a renormalizable scale invariant term involving the square of the curvature of the world sheet, thus arriving at the theory of rigid strings. In the large N approximation, where N is the dimension of space- time, this model does not undergo a phase transition. One way to induce such a phase transition [2] is to couple the rigid string to the long range Kalb-Ramond field [3]. It has been shown that to leading and subleading order the resulting theory does indeed undergo a phase transition to a region of long range order [2].

One of the aims of the present work is to provide a framework within which such an analysis can be carried out for standard superstrig theories. As mentioned above, the two main ingredients of such a framework are a scale invariant addition to the superstring action and the coupling of a long range interaction to the superstrings characterized by a dimensionless parameter. Once such a space-time scale invariant superstring action is constructed, it can also be cosidered as a superstring theory in its own right. As will be seen below, it may provide a dynamical basis for understanding the origin of the string tension. So, we will construct such superstring actions without reference to a particular application. Not having an a priori knowledge of the structure of a space-time scale invariant superstring action, it turns out to be more convenient to first construct its coupling to the long range field. As will be seen below, the regularization of this interaction will lead us directly to the sought after superstring action.

A prime candidate for the long range interaction is a gauge field. Although it is possible to construct interactions between strings and gauge fields which are characterized by a dimensionful coupling constant [4], it can be easily seen that there are no such dimensionless interaction terms with at most two derivatives between strings and non-supersymmetric gauge theories. The only exception is the parity violating expression

$$ie \int d\sigma F^* \ . \tag{1}$$

In (1) $d\sigma$ is the surface measure, and F^* is the dual of the gauge field strength. Therefore, it is of interest to explore whether supersymmetric gauge theories can couple to a superstring theory with a dimensionless coupling strength. In the interest of clarity, in this work we confine ourselves to the abelian case. The non-abelian version will be discussed elsewhere [5].

In a recent work [6], we showed that it is possible to extend the notion of Wilson loop to supersymmetric gauge theories. This was accomplished by making use of the Stoke's theorem, and its non- abelian version, to express the Wilson loop in terms of the field strength on a two-suface with a boundary and then seek its supersymmetric generalization. The result was expressed in terms of chiral superfields and chiral currents on the two-surface. The expression for the chiral current necessary for this supersymmetrization turned out to be not the most general one that could be constructed on the two surface. Its genralization led us to the construction of a new "stringy" observable which depends on the metric and which has no analogue in non-supersymmetric gauge theories. It is also characterized by a dimensionless coupling constant.

To write down and study the properties of this stringy observable, we follow the notation and conventions of reference [6] and use the standard two component superspace formalism [7]. Thus the components of the supersymmetric vielbein are given by

$$\begin{aligned} v_a^{\alpha\dot{\alpha}} &= \partial_a x^{\alpha\dot{\alpha}}(\xi) - \frac{i}{2}(\theta^\alpha(\xi)\partial_a\theta^{\dot{\alpha}}(\xi) + \theta^{\dot{\alpha}}(\xi)\partial_a\theta^\alpha(\xi)) \\ v_a^\alpha &= \partial_a\theta^\alpha(\xi) \\ v_a^{\dot{\alpha}} &= \partial_a\theta^{\dot{\alpha}}(\xi). \end{aligned} \tag{2}$$

They are invariant under global space-time supersymmetry transformation rules defined

$$\begin{aligned} \delta x^{\alpha\dot{\alpha}}(\xi) &= \frac{i}{2}(\epsilon^\alpha\theta^{\dot{\alpha}}(\xi) + \epsilon^{\dot{\alpha}}\theta^\alpha(\xi)) \\ \delta\theta^\alpha(\xi) &= \epsilon^\alpha \\ \delta\theta^{\dot{\alpha}}(\xi) &= \epsilon^{\dot{\alpha}}. \end{aligned} \tag{3}$$

The requirement that, e.g, in four dimensions, the coordinates θ satisfy the Majorana condition demands that ϵ be a Majorana. Thus we define:

$$\begin{aligned} C_{ab}^\alpha &= v_a^{\alpha\dot{\alpha}} v_{b\dot{\alpha}} \\ C_{ab}^{\dot{\alpha}} &= v_a^{\alpha\dot{\alpha}} v_{b\alpha}. \end{aligned} \tag{4}$$

The field content of the supersymmetric Maxwell theory can be expressed in terms of the chiral superfield $W_\alpha(x,\theta)$ and its conjugate. They satisfy the chirality conditions:

$$\begin{aligned} D_\alpha W_{\dot{\beta}} &= D_{\dot{\alpha}} W_\beta = 0 \\ D^{\dot{\alpha}} W_{\dot{\alpha}} &= D^\alpha W_\alpha. \end{aligned} \tag{5}$$

The W's are determined in terms of an unconstrained vector superfield V:

$$W_\alpha = \frac{-i}{2}\bar{D}^2 D_\alpha V \quad ; \quad W_{\dot{\alpha}} = \frac{i}{2} D^2 D_{\dot{\alpha}} V \tag{6}$$

which are solutions of (5). The $W's$ are invariant under the gauge transformation

$$\delta V = i(\bar{\Lambda} - \Lambda) \tag{7}$$

where Λ ($\bar{\Lambda}$) is a chiral (anti-chiral) parameter superfield. The component expansion of V and W_α in the Wess-Zumino gauge are respectively,

$$V = (0, 0, 0, 0, A_{\alpha\dot{\alpha}}, \psi_\alpha, \psi_{\dot{\alpha}}, D) \tag{8a}$$

$$W_\alpha = \psi_\alpha - \theta^\beta f_{\alpha\beta} - i\theta_\alpha D + \frac{i}{2}\theta^2 \partial_{\alpha\dot{\alpha}}\psi^{\dot{\alpha}} \tag{8b}$$

where ψ is the superpartner of the gauge field $A_{\alpha\dot{\alpha}}$, $f_{\alpha\beta} = \frac{1}{2}\partial_{(\alpha\dot{\alpha}}A^{\dot{\alpha}}_{\beta)}$ is the Maxwell's field strength and D is an auxiliary field. An important property of the W_α ($W_{\dot{\alpha}}$) is that it is invariant under the chiral part (anti-chiral part) of the supersymmetry transformations of the component fields:

$$\delta_{\epsilon^\alpha} W_\alpha = \delta_{\epsilon^{\dot{\alpha}}} W_{\dot{\alpha}} = 0\ . \tag{9a}$$

$$\begin{aligned} \delta A^{\alpha\dot{\alpha}} &= i(\epsilon^\alpha \psi^{\dot{\alpha}} + \epsilon^{\dot{\alpha}}\psi^\alpha) \\ \delta\psi_\alpha &= \epsilon^\beta f_{\alpha\beta} + i\epsilon_\alpha D \\ \delta D &= \frac{1}{2}\partial_{\alpha\dot{\alpha}}(\epsilon^\alpha\psi^{\dot{\alpha}} - \epsilon^{\dot{\alpha}}\psi^\alpha) \end{aligned} \tag{9b}$$

With these preliminaries, we can now write down the stringy interaction obtained in reference [6] :

$$S_{int.} = \kappa \int_\Sigma d^2\xi \sqrt{-h} h^{ab} C^\alpha_{ab}(\xi) W_\alpha(x(\xi), \theta(\xi)) + h.c \tag{10}$$

where h^{ab} is the metric on the two surface, h is its determinant, and $C^\alpha_{ab}(\xi)$ are the components of a manifestly supersymmetric invariant spinor tensor given in (4). The quantities $W_\alpha(x(\xi), \theta(\xi))$ are the supersymmetric invariant abelian chiral superfields given above, and h.c denotes hermitian conjugation. The interaction (10) is also invariant under the local scale transformation $h_{ab} \rightarrow \Lambda(\xi) h_{ab}$ of the world sheet metric.

We know of no way of constructing this interaction term in the absence of supersymmetry. It is space-time supersymmetric, gauge invariant, reparemetrization invariant, and locally scale invariant on the world sheet. It is also characterized by a new coupling constant κ which is, at least classically, different from the gauge coupling e. In four space-time dimensions, e is dimensionless. So is κ . in space-time dimensions three, six, and ten, in which supersymmetric gauge theories exist also, κ is not dimensionless but has the same dimension as e . It is, however, possible to modify the action (10) so that κ remains dimensionless in any of these dimesions.

In analogy with the expression for the supersymmetric Wilson loop, we can now define a new supersymmetric and gauge invariant "stringy" observable

$$\Psi(\Sigma) = e^{S_{int.}} \tag{11}$$

If we take the surface Σ to have a boundary, a closed loop C, then the correlation functions of $\Psi(\Sigma)$ might be useful for describing loops formed by pair creation and annihilation of superparticles, particularly in strongly coupled super QED. As mentioned above, the superstring-like observable can be expressed in terms of chiral currents on the surface. Define

$$\mathcal{J}^{\alpha}(z) = \kappa \int_{\Sigma(C)} d^2\xi \mathcal{Q}^{\alpha}(\xi)\delta^6(z - z(\xi)) \tag{12a}$$

where $\delta^6(z - z(\xi))$ is the chiral delta function which in the chiral representation takes the form $= \delta^4(x - x(\xi))(\theta - \theta(\xi))^2$, and

$$\mathcal{Q}^{\alpha}(\xi) = \sqrt{-h}h^{ab}C^{\alpha}_{ab}(\xi) \; . \tag{12b}$$

The action (10) can then be rewritten as [6] :

$$S_{int} = \int d^6z(\mathcal{J}^{\alpha}W_{\alpha} + h.c) \; . \tag{13}$$

In this form, it is manifestly supersymmetric, and gauge invariant.

From equation (12b) it is clear why the stringy observable has no counter part in non-supersymmetric gauge theories. In the absence of supersymmetry, $C^{\alpha}_{ab} = 0$, and we loose the superstring-gauge field interaction (10).

Next, we compute the expectation value, $< \Psi(C) >$, of the stringy observable. Since the theory is abelian one can easily show that [8]:

$$< \Psi(\Sigma) >= e^{-\frac{1}{4}\int d^6z \int d^6z'(\mathcal{J}^{\alpha}(z)(\mathcal{J}^{\beta}(z')<W_{\alpha}(z)W_{\beta}(z')>+h.c} \; . \tag{14}$$

The average in (14) is taken with respect to the Boltzmann factor of the super-Maxwell action

$$S_{Super\ Maxwell} = \int d^6zW^{\alpha}(z)W_{\alpha}(z) \tag{15}$$

Using the expression of W_{α} in (8b) one finds:

$$< W_{\alpha}(z)W_{\beta}(z') >= \frac{1}{2}\delta^6(z - z')\epsilon_{\alpha\beta} \tag{16}$$

Inserting this into (14) and doing one of the z integrations we obtain

$$S_{string} = -\frac{\kappa^2}{8}\int_{\Sigma} d^2\xi \int_{\Sigma} d^2\xi'\delta^4(x(\xi) - x(\xi'))(\theta(\xi) - \theta(\xi'))^2\mathcal{Q}^{\alpha}(\xi)\mathcal{Q}_{\alpha}(\xi') + h.c \; . \tag{17}$$

Clearly (17) is divergent and requries regularization. We will regularize the delta function by making the replacement

$$\delta^4(x(\xi) - x(\xi')) \to \delta^4_{\epsilon}(x(\xi) - x(\xi')) = \frac{1}{\pi^2\epsilon^4}e^{\frac{(x(\xi)-x(\xi'))^2}{\epsilon^2}} \tag{18}$$

and in the end taking the limit $\epsilon \to 0$. This regularization method can be interpreted geometrically by the manifold splitting regularization method where one displaces Σ in the first measure infinitesimally away that in the second measure along some unit normal $n^\mu(\xi)$. Thus we define Σ_ϵ to have coordinates $y^\mu = x^\mu + \epsilon n^\mu$ where x^μ is the coordinate on Σ. Inserting (18) into (17) and taking the limit $\epsilon \to 0$ we obtain the following action:

$$S = -\frac{\kappa^2}{8\pi}\int_\Sigma d^2\xi\sqrt{-h}(\frac{h}{G})^{\frac{1}{2}}G^{ab}K_{ab} \tag{19}$$

where G^{ab} is the inverse of the induced metric G_{ab} on the world sheet:

$$G_{ab} = v_a^\mu v_b^\mu \eta_{\mu\nu} \tag{20}$$

$$K_{ab} = K^\alpha K_\alpha g_{ab} + h.c\ . \tag{21}$$

$$K^\alpha = h^{ab}C_{ab}^\alpha(\xi), \quad g_{ab} = v_a^\alpha v_{b\alpha}$$

It is straight forward to show that

$$K_{ab} = -2K_aK_b \tag{22a}$$

where

$$K_a = K^\alpha v_{a\alpha}\ . \tag{22b}$$

Further manpulations makes the action takes the simple form

$$S = \frac{\kappa^2}{4\pi}\int_\Sigma d^2\xi\sqrt{-h}(\frac{h}{G})^{\frac{1}{2}}(G^{ab} - G^{cd}h_{cd}h^{ab})t_at_b \tag{23a}$$

where

$$t_a = \frac{1}{2}v_a^{\alpha\dot{\alpha}}e_{\alpha\dot{\alpha}}\ . \tag{23b}$$

$$e^{\alpha\dot{\alpha}} = \epsilon^{ab}\partial_a v_b^{\alpha\dot{\alpha}}$$

The quantity ϵ is the covariant antisymmetric tensor on the world sheet. To put our action in a more familiar form, we resort to the 4-component notation, with $\mu = 0, ...D-1$ being the space-time index, and $\alpha = 1, ...4$ the spinor index. We have

$$S = \frac{\kappa^2}{16\pi}\int_\Sigma d^2\xi\sqrt{-h}(\frac{h}{G})^{\frac{1}{2}}(G^{ab} - G^{cd}h_{cd}h^{ab})v_a^\mu v_b^\nu e_\mu e_\nu \tag{24a}$$

where

$$v_a^\mu = \partial_a x^\mu(\xi) - i\bar{\theta}^\alpha(\xi)\Gamma^\mu\partial_a\theta_\alpha(\xi) \tag{24b}$$

$$e^\mu = \epsilon^{ab}\partial_a v_b^\mu$$

The space-time vectors e^μ are supersymmetric and are of mass dimension. The supersymmetric action (24a) is invariant under the two dimensional general coordinate

transformations, space-time supersymmetry transformations, global space-time scale transformations, and local world sheet scale transformations given by :

$$h_{ab} \to \Lambda(\xi) h_{ab}, \quad e_\mu \to \Lambda^{-1}(\xi) e_\mu \ . \tag{25}$$

Except for space-time scale invariance, these invariances are shared with the Green-Schwarz action [9]. So, the distinguishing feature of our superstring action is its invariance under space- time scale transformations.

The action (24a) can be further simplified. Define the set of supersymmetric, locally scale invariant vectors:

$$\sigma^\mu = \frac{\varepsilon^{ab}}{(G)^{\frac{1}{2}}} \partial_a v_b^\mu \tag{26}$$

where ε is the numerical antisymmetric tensor that transforms as a density ($\varepsilon^{ab} = \sqrt{-h}\epsilon^{ab}$). Then the action (24a) becomes

$$S = \frac{\kappa^2}{16\pi} \int_\Sigma d^2\xi \sqrt{G} (G^{ab} - G^{cd} h_{cd} h^{ab}) v_a^\mu v_b^\nu \sigma_\mu \sigma_\nu \ . \tag{27}$$

The above action is expressed in a second order formalism, and it is more desirable to express it in its first order form. Since we have established all the invariances of the action (27), in particular its space-time scale invariance, we can simply ask if there are any actions consistent with these symmetries. We find that there are only two such actions. One is given by

$$S_0 = \frac{\kappa_0^2}{16\pi} \int_\Sigma d^2\xi \sqrt{-h} h^{ab} v_a^\mu v_b^\nu \sigma_\mu \sigma_\nu \ . \tag{28}$$

It is the first order form of the action (27). To write down the other action we first define the following composite field:

$$\Phi = \sigma^\mu \sigma_\mu \ . \tag{29}$$

Then we have

$$S_1 = \frac{\kappa_1^2}{4\pi} \int_\Sigma d^2\xi \sqrt{-h} h^{ab} G_{ab} \Phi \ . \tag{30}$$

We note in passing that an appropriate power of the operator Φ can also be used in the interaction term (10) so as to make the coupling constant dimensionless in all the relevant space-time dimensions.

The full action of our space-time scale invariant superstring theory is given by

$$S_{superstring} = S_0 + S_1 \ . \tag{31}$$

At this stage the two couplings in this expression are independent. But when coupled to supersymmetric gauge theories as in (10), it is expected that quantum loop calculations relate these couplings to that of the gauge theory. The equation resulting

from the variation of the action (31) with respect to h_{ab} leads to the usual vanishing of the components of the energy-momentum tensor:

$$\Xi_{ab} - \frac{1}{2} h_{ab} h^{cd}\, \Xi_{cd} = 0 \tag{32}$$

$$\Xi_{ab} = \frac{1}{4\pi}(\kappa_0^2 \tau_a \tau_b + \kappa_1^2 G_{ab} \Phi)$$

$$\tau_a = \frac{1}{2} \upsilon_a^\mu \sigma_\mu .$$

Let us now consider possible applications of the above formalism. One possibility to which we have alluded above is to add the space-time scale invariant action (31) and the long range interaction (10) to one of the existing superstring actions and study the resulting theory to see if it has a (super) QCD-like phase. One would expect that this theory would be more appropriate for such an analysis than its bosonic version. The other possibility is to consider the action (31) as a superstring theory in its own right. In that case, we note that the structure of the action (30) suggests an interesting possibility. Aside from the composite operator Φ, it is identical to the Green-Schwarz action [9]. If due to the unknown dynamics at the Planck scale this four fermion operator acquires a non-vanishing expectation value, i.e., if

$$\frac{\kappa_1^2}{4\pi} < \Phi >= \mu_{string\ tension} \tag{31}$$

then we have a dynamical basis for understanding how a dimensional parameter arises in superstring theories. Clearly, much remains to be done.

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